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PROPAGATION OF PLANE WAVES OF A DEFECT FIELD IN A VISCOPLASTIC MEDIUM IN THE PRESENCE OF AN INTERFACE BETWEEN TWO MEDIA

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The dynamic equations of the continual theory of defects are used to study the structure of the waves of a defect field characterized by a defect density tensor and a defect flux tensor in a viscoplastic medium. Relations are obtained that define the passage of defect field waves through an interface between two media. Particular cases of media with rapidly and slowly decaying waves are considered.

Key words: *the continual theory of defects, transverse waves, decay, reflection, refraction.*

Introduction. Wave processes is an important form of motion of matter. Wave propagation is the fastest mode of energy transfer that implements a transition from nonequilibrium to equilibrium in a system [1]. In different material objects, the mechanisms of perturbation propagation are considerably different. Defect field waves propagate because a variable dislocation density that arises at any point of the body being deformed generates a defect flow at neighboring points.

Continuing the studies of [2], where the propagation of plane defect waves in a viscoplastic medium is considered on the basis of defect field theory, we take into account the effect of an interface between two media. The results obtained in [3–5] suggest a special role of interfaces in deformation processes; therefore, the behavior of loaded materials in the presence of an interface is an important problem of the mechanics of deformable bodies. Recently, this problem has been the subject of much research (see, for example, [6, 7]).

The present paper gives the equations of defect field theory for the case of a viscoplastic body. The wave structure of a defect field characterized by a defect density tensor and a defect flux tensor is analyzed. This analysis is of great significance for a number of reasons. First, investigation of the structure of defect field waves is a necessary step in studying the propagation of plane defect waves at an interface of two media. Second, the analysis performed supplements experimental measurements of plastic distortion rates [8] by corresponding components of the dislocation density tensor, which characterizes the defect structure of material. That is, these results allow the dislocation density related to couple stresses to be evaluated from plastic strain rates measured in experiments [9].

This paper studies the propagation of plane defect waves in the presence of an interface between two media. The relations obtained (refraction law and reflection and refraction coefficients) are used to analyze waves propagating in media with rapid and slow decay; the general case of wave propagation in media with arbitrary decay is also considered.

1. Formulation of the Problem. The basic dynamic equations of defect field theory [1, 10] are written as

$$B(\nabla\cdot I)=-\boldsymbol{P},\qquad \nabla\cdot\alpha=0,$$
$$\frac{\partial\alpha}{\partial t}=\nabla\times I,\qquad S(\nabla\times\alpha)=-B\,\frac{\partial I}{\partial t}-\sigma, \tag{1}$$

where α and I are the density and dislocation flux tensors, $\boldsymbol{P}$ and σ are the effective stresses and momentum, B and S are constants of the theory, and the dot and multiplication signs denote scalar and vector products, respectively. Equations (1) imply that the effective stresses and momentum satisfy the compatibility equation

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$$\frac{\partial \boldsymbol{P}}{\partial t} = \nabla \cdot \sigma, \tag{2}$$

which is the equation of dynamic equilibrium in continuum mechanics. According to the definition of a viscoplastic body [11], which implies the dependence of stresses on the plastic strain rate, we assume

$$\sigma = \eta I. \tag{3}$$

Here η is the viscosity tensor and I is the defect flux tensor determined by the plastic distortion rate β [12]:

$$I = -\frac{\partial \beta}{\partial t}.$$

In this case, $\alpha = -\nabla \times \beta$. As noted above [2], relation (3) can be written formally on the basis of the analogy between Eqs. (1) and the Maxwell electrodynamics equations [13]. In the general case [14], the effective stresses have the form

$$\sigma = \sigma^{\text{ext}} + \sigma^{\text{int}} + \sigma^{\text{vis}}.$$

In the model, expression (3) implies that $\sigma^{\text{ext}} \ll \sigma^{\text{vis}}$ and $\sigma^{\text{int}} \ll \sigma^{\text{vis}}$ (σ^{ext} are the stresses due to the external load, σ^{int} are the internal stresses determined by material defects, and σ^{vis} are viscous stresses).

Considering Eqs. (1)–(3) together, one can show that in a viscoplastic medium, the effective momentum decreases with time as

$$\boldsymbol{P} = \boldsymbol{P}_0 \exp(-t/\tau), \tag{4}$$

where $\tau = B/\eta$ is the relaxation time and $\boldsymbol{P}_0$ is the initial momentum. The decrease in the momentum is the faster, the smaller the relaxation time determined by the parameters B and η, which characterize the inertia of the ensemble of defects and the viscosity of the medium. According to (4), a defect field in a viscoplastic medium cannot be responsible for the occurrence of momentum because if $\boldsymbol{P}(0) = 0$, then, in the following, $\boldsymbol{P}(t) = 0$. With allowance for equalities (3) and (4), the original system (1) is written as

$$\begin{gathered} \nabla \cdot I = 0, \qquad \nabla \cdot \alpha = 0, \\ \frac{\partial \alpha}{\partial t} = \nabla \times I, \qquad S(\nabla \times \alpha) = -B\,\frac{\partial I}{\partial t} - \eta I. \end{gathered} \tag{5}$$

For completeness of system (5), which allows defect field characteristics to be uniquely determined from specified initial values, one needs to specify boundary conditions for α and I on the interface. These boundary conditions can be obtained by a known method [15] using the integrated form of Eqs. (5). We assume that the normal components α_{nk} and I_{nk} and the tangential components α_{tk} and I_{tk} of the defect field characteristics on the interface satisfy the conditions

$$\begin{gathered} I^1_{nk} - I^2_{nk} = 0, \qquad \alpha^1_{nk} - \alpha^2_{nk} = 0, \\ I^1_{tk} - I^2_{tk} = 0, \qquad S_1\alpha^1_{tk} - S_2\alpha^2_{tk} = 0. \end{gathered} \tag{6}$$

2. Structure of Defect Field Waves. We consider a defect field in which the quantities α and I depend on one dimensional coordinate $\xi = \boldsymbol{m} \cdot \boldsymbol{r}$ and time t. In this case, each of nine components α_{ik} satisfies one scalar equation

$$\frac{B}{S}\,\frac{\partial^2 \alpha_{ij}}{\partial t^2} - \frac{\partial^2 \alpha_{ij}}{\partial \xi^2} + \frac{\eta}{S}\,\frac{\partial \alpha_{ij}}{\partial t} = 0. \tag{7}$$

A similar equation is written for the components I_{ik}. As shown in [2], Eqs. (7) describe the propagation of two plane harmonic waves

$$\alpha^{1,2} = \alpha \exp(i\omega(t \pm \xi/V)), \qquad I^{1,2} = I \exp(i\omega(t \pm \xi/V)) \tag{8}$$

at a velocity

$$V = \sqrt{(S/B)/(1 + i\eta/(B\omega))}.$$

To determine the structure of the defect field waves, we consider a wave propagating in the direction $\boldsymbol{m}$. In this case, Eqs. (5) become

$$\frac{\partial}{\partial \xi}(\boldsymbol{m}I) = 0, \qquad \frac{\partial}{\partial \xi}(\boldsymbol{m}\alpha) = 0,$$
$$\frac{\partial}{\partial \xi}[\boldsymbol{m}I] = \frac{\partial \alpha}{\partial t}, \qquad S\frac{\partial}{\partial \xi}[\boldsymbol{m}\alpha] = -B\frac{\partial I}{\partial t} - \eta I. \tag{9}$$

From the first two equations it follows that $\partial_\xi I_{\xi i} = 0$ and $\partial_\xi \alpha_{\xi i} = 0$; i.e, if the projections of the tensors α and I onto the direction of wave propagation are not equal to zero, they can depend only on time. Multiplying the last two equations scalarly by the vector $\boldsymbol{m}$, we obtain

$$\frac{\partial \alpha_{\xi i}}{\partial t} = 0, \qquad B\frac{\partial I_{\xi i}}{\partial t} + \eta I_{\xi i} = 0. \tag{10}$$

The first of these equations implies that the projection of the defect density wave onto the direction ξ also does not depend on time; i.e., $\alpha_{\xi i} \equiv 0$. In other words, the defect density wave is transverse and all nonzero components lie in the plane of the wave. From the second equation of system (10), it follows that the longitudinal components of the defect flux tensor decrease with time as

$$I_{\xi i} = I_{\xi i}(0)\exp(-t/\tau);$$

therefore, the wave of the tensor I is also transverse in a viscoplastic medium.

Let us determine the relationship between the defect field characteristics in a plane harmonic wave. Substituting (8) into the third equation of system (9), we obtain

$$\alpha = [\boldsymbol{m}I]/V. \tag{11}$$

The parameter

$$1/V = \sqrt{B/(S(1+i\eta)/(B\omega))} = \sqrt{1+i\tan\delta}/C = (n+i\chi)/C, \tag{12}$$

which defines the quantitative relationship between α and I, has the meaning of the impedance of the medium. Here $\tan\delta = \eta/(B\omega)$ is the loss factor, n and χ are the refraction and absorption factors, and $C = \sqrt{S/B}$.

Using (11), we obtain the ratio of the absolute values of the defect field characteristics and the phase shift φ:

$$|I|/|\alpha| = |V| = C/\sqrt{n^2+\chi^2}, \qquad \tan\varphi = \chi/n = \tan(\delta/2), \quad \varphi = \delta/2.$$

Here $n = \sqrt{(\sqrt{\tan^2\delta+1}+1)/2}$ and $\chi = \sqrt{(\sqrt{\tan^2\delta+1}-1)/2}$ [2].

Let us consider the limiting cases where slowly and rapidly decaying waves propagate in the medium. For slowly decaying waves, $\tan\delta \ll 1$; therefore, $n = 1$, $\chi = (\tan\delta)/2$, and

$$|I|/|\alpha| = |V| = C/\sqrt{1+(\tan^2\delta)/4} \approx C, \qquad \tan\varphi = (\tan\delta)/2.$$

For rapidly decaying waves, $\tan\delta \gg 1$, $n \approx \chi = \sqrt{(\tan\delta)/2}$, and

$$|I|/|\alpha| = |V| \approx C/\sqrt{\tan\delta}, \qquad \tan\varphi \approx 1.$$

However, in the case $\tan\delta \gg 1$, the wave process practically does not occur because the defect wave decays at very small distances from the boundary. The amplitude decreases by a factor of e at a distance

$$d = C/(\chi\omega) = \lambda/(2\pi\chi), \tag{13}$$

which, at $\tan\delta \gg 1$ and $n \approx \chi \gg 1$, is much smaller than the wavelength λ.

3. Passage of a Plane Defect Wave through an Interface of Two Media. 3.1. *Reflection and Refraction Laws.* We assume that the interface of two homogeneous media coincides with the plane $z = 0$ in a Cartesian coordinate system. The media located above ($z > 0$) and below ($z < 0$) the interface are characterized by the parameters B_1, S_1, and η_1 and B_2, S_2, and η_2, respectively. A plane wave from medium 1 is incident on the interface at an angle θ_0 to the z axis with a frequency ω and a wave vector $\boldsymbol{K}_0 = k_1\boldsymbol{m}_0$, where $k_1 = \omega/V_1$ and $\boldsymbol{m}_0$ is the unit normal vector to the incident wave front (Fig. 1). The incidence plane containing the vector $\boldsymbol{K}_0$ and the z axis is made coincident with the plane xz. The wave vector of the reflected wave is denoted by $\boldsymbol{K}_1 = k_1\boldsymbol{m}_1$, and the wave vector of the refracted wave by $\boldsymbol{K}_2 = k_2\boldsymbol{m}_2$; $\boldsymbol{z}_0$ is the unit normal vector to the boundary directed from medium 2 to medium 1.

According to (8) and (11), the defect fields can be written as follows:
for the incident wave,

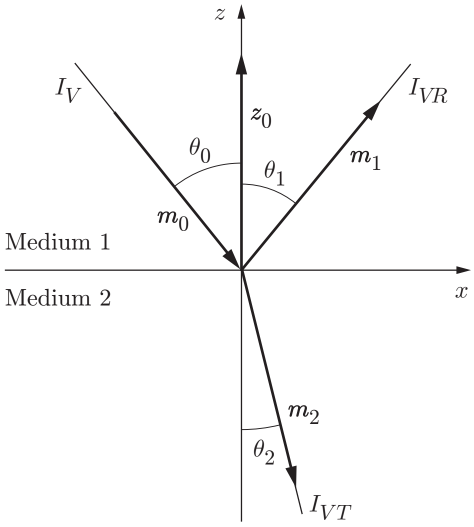

Fig. 1. Reflection and refraction of a plane wave at an interface of two media.

$$I = I_0 \exp(-i\omega t + ik_1 \boldsymbol{m}_0 \boldsymbol{r}), \qquad \alpha = [\boldsymbol{m}_0 I_0] Z_1 \exp(-i\omega t + ik_1 \boldsymbol{m}_0 \boldsymbol{r}); \tag{14}$$

for the reflected wave,

$$I_R = I_1 \exp(-i\omega t + ik_1 \boldsymbol{m}_1 \boldsymbol{r}), \qquad \alpha_R = [\boldsymbol{m}_1 I_1] Z_1 \exp(-i\omega t + ik_1 \boldsymbol{m}_1 \boldsymbol{r});$$

and for the refracted wave,

$$I_T = I_2 \exp(-i\omega t + ik_2 \boldsymbol{m}_2 \boldsymbol{r}), \qquad \alpha_T = [\boldsymbol{m}_2 I_2] Z_2 \exp(-i\omega t + ik_2 \boldsymbol{m}_2 \boldsymbol{r}). \tag{15}$$

Here $Z_1 = 1/V_1$ and $Z_2 = 1/V_2$ are the impedances of medium 1 and 2, respectively.

At $z = 0$, boundary conditions (6) for the tangential component of the total wave field α and I should be satisfied:

$$\begin{gathered} [\boldsymbol{z}_0 I_0]\, \mathrm{e}^{i\boldsymbol{K}_0\boldsymbol{r}} + [\boldsymbol{z}_0 I_1]\, \mathrm{e}^{i\boldsymbol{K}_1\boldsymbol{r}} = [\boldsymbol{z}_0 I_2]\, \mathrm{e}^{i\boldsymbol{K}_2\boldsymbol{r}}, \\ [\boldsymbol{z}_0[m_0 I_0]]\, \mathrm{e}^{i\boldsymbol{K}_0\boldsymbol{r}} + [\boldsymbol{z}_0[m_1 I_1]]\, \mathrm{e}^{i\boldsymbol{K}_1\boldsymbol{r}} = (S_2 Z_2/S_1 Z_1)[\boldsymbol{z}_0[m_2 I_2]]\, \mathrm{e}^{i\boldsymbol{K}_2\boldsymbol{r}}\,. \end{gathered} \tag{16}$$

Relations (16), which are valid at all points of the plane $z = 0$, imply identical dependences of the three wave fields on the coordinates x and y at $z = 0$; i.e., the phase factors should satisfy the conditions

$$k_1 \boldsymbol{m}_0 \boldsymbol{r}\Big|_{z=0} = k_1 \boldsymbol{m}_1 \boldsymbol{r}\Big|_{z=0} = k_2 \boldsymbol{m}_2 \boldsymbol{r}\Big|_{z=0}$$

or

$$k_1 \sin\theta_0 = k_1 \sin\theta_1 = k_2 \sin\theta_2, \tag{17}$$

where θ_1 is the reflection angle and θ_2 is the refraction angle (Fig. 1). From (17) it follows that the reflection angle is equal to the incidence angle (reflection law):

$$\theta_0 = \theta_1,$$

and the sines of the refraction and incidence angles are linked by the relation

$$\sin\theta_2/\sin\theta_0 = k_1/k_2, \tag{18}$$

which is an analog of the refraction law or Snell's law in electrodynamics [1].

3.2. *Reflection and Refraction Coefficients.* To determine the amplitudes of the reflected and refracted waves, we revert to system (16). We consider waves of two different linear polarizations: a horizontally polarized wave with tensor I, whose nonzero components are perpendicular to the incidence plane ($I_{xi} = I_{zi} = 0$ and $I_{yi} \neq 0$), and a vertically polarized wave with tensor I, whose components belong to the incidence plane ($I_{yi} = 0$, $I_{xi} \neq 0$, and $I_{zi} \neq 0$). In the first case (for horizontally polarized waves) from Eqs. (16) for unknown amplitudes I_1 and I_2, we obtain

$$I_0 + I_1 = I_2, \qquad S_1 Z_1 (I_0 - I_1)\cos\theta_0 = S_2 Z_2 I_2 \cos\theta_2. \tag{19}$$

Solving (19), we find the coefficients linking the amplitudes of the reflected and refracted waves to the amplitude of the incident wave:

$$R_g = \frac{S_1 Z_1 \cos\theta_0 - S_2 Z_2 \cos\theta_2}{S_1 Z_1 \cos\theta_0 + S_2 Z_2 \cos\theta_2}, \qquad T_g = \frac{2 S_1 Z_1 \cos\theta_0}{S_1 Z_1 \cos\theta_0 + S_2 Z_2 \cos\theta_2}. \tag{20}$$

Here $R_g = I_1/I_0$ and $T_g = I_2/I_0$. For vertically polarized waves, system (16) becomes

$$(I_0 - I_1)\cos\theta_0 = I_2 \cos\theta_2, \qquad S_1 Z_1 (I_0 + I_1) = S_2 Z_2 I_2.$$

The coefficients linking the amplitudes of the three waves, known in electrodynamics as Fresnel coefficients, are written as

$$R_v = \frac{S_2 Z_2 \cos\theta_0 - S_1 Z_1 \cos\theta_2}{S_2 Z_2 \cos\theta_0 + S_1 Z_1 \cos\theta_2}, \qquad T_v = \frac{2 S_1 Z_1 \cos\theta_0}{S_2 Z_2 \cos\theta_0 + S_1 Z_1 \cos\theta_2}. \tag{21}$$

Using (18), we write expressions (20) and (21) as follows:

$$\begin{gathered} R_g = \frac{(S_1 V_2/S_2 V_1)\cos\theta_0 - \sqrt{1-(V_2/V_1)^2 \sin^2\theta_0}}{(S_1 V_2/S_2 V_1)\cos\theta_0 + \sqrt{1-(V_2/V_1)^2 \sin^2\theta_0}}, \\ T_g = \frac{(2 S_1 V_2/S_2 V_1)\cos\theta_0}{(S_1 V_2/S_2 V_1)\cos\theta_0 + \sqrt{1-(V_2/V_1)^2 \sin^2\theta_0}}; \end{gathered} \tag{22}$$

$$\begin{gathered} R_v = \frac{(S_2 V_1/S_1 V_2)\cos\theta_0 - \sqrt{1-(V_2/V_1)^2 \sin^2\theta_0}}{(S_2 V_1/S_1 V_2)\cos\theta_0 + \sqrt{1-(V_2/V_1)^2 \sin^2\theta_0}}, \\ T_v = \frac{2\cos\theta_0}{(S_2 V_1/S_1 V_2)\cos\theta_0 + \sqrt{1-(V_2/V_1)^2 \sin^2\theta_0}}. \end{gathered} \tag{23}$$

For normal incidence($\theta_0 = 0$),

$$R_g = \frac{S_1 Z_1 - S_2 Z_2}{S_1 Z_1 + S_2 Z_2} = -R_v.$$

4. Analysis of Particular Cases. 4.1. *Slowly Decaying Waves.* Let us analyze the general expressions (20) and (21) for the particular cases of limiting media in which rapidly and slowly decaying waves propagate. The following preliminary remarks should be made. The interface of two media is actually a thin transitional layer rather than a geometrical surface. Formulas (17) are valid irrespective of any assumptions on the nature of this layer. The derivation of formulas (22) and (23) using the conditions on the interface assumes that the thickness of the transitional layer δ is small compared to the wavelength λ. Usually, in a macroscopic description, $\lambda \gg \delta$.

We consider media in which waves decay slowly ($\tan\delta_1 \ll 1$ and $\tan\delta_2 \ll 1$) and, hence, the ratio

$$\frac{V_2}{V_1} = \frac{C_2}{C_1}\sqrt{\frac{1 + i\tan\delta_1}{1 + i\tan\delta_2}} \simeq \frac{C_2}{C_1} = \sqrt{\frac{S_2 B_1}{S_1 B_2}} \tag{24}$$

is real. For $V_2/V_1 < 1$, the reflection and refraction coefficients (22) and (23) are also real; i.e., the phase shift between the incident and reflected waves is 0 or π.

Figure 2 gives curves of $R_g(\theta)$ and $R_v(\theta)$ for different ratios of the model parameters. The curve of $R_g(\theta)$ has no singularities and vanishes only in the case of $V_2/V_1 = 1$ and $S_1/S_2 = 1$, where the properties of both media are identical; i.e, the reflection disappears when the interface disappears. The reflection coefficient $R_v(\theta)$ defined as

$$R_v = \frac{S_2 \sin\theta_0 \cos\theta_0 - S_1 \sin\theta_2 \cos\theta_2}{S_2 \sin\theta_0 \cos\theta_0 + S_1 \sin\theta_2 \cos\theta_2} = \frac{\tan(\theta_0 - \varphi)}{\tan(\theta_0 + \varphi)},$$

has two singularities: singularity for $V_2/V_1 = 1$ and $S_1/S_2 = 1$ and singularity for

$$\theta_0 + \varphi = \pi/2. \tag{25}$$

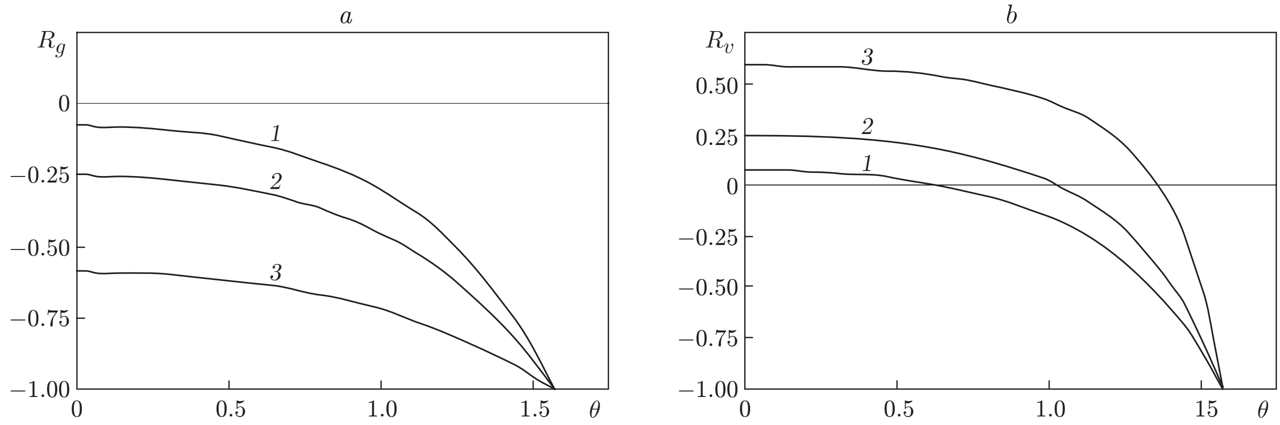

Fig. 2. Reflection coefficients of the defect flux tensor components perpendicular (a) and parallel (b) to the incidence plane versus the incidence angle for $V_2/V_1 = 0.6$: $S_1/S_2 = 1.43$ (1), 1 (2), and 0.43 (3).

Here $\varphi = (1/2)\arcsin((S_1/S_2)\sin 2\theta_2)$. A simultaneous consideration of relations (18) and (25) yields the incidence angle at which the second singularity takes place:

$$\theta_0^* = \arcsin\sqrt{\frac{(V_2/V_1)^2 - (S_2/S_1)^2}{(V_2/V_1)^4 - (S_2/S_1)^2}}.$$

The angle θ_0^* is the angle of complete polarization (Brewster angle in electrodynamics) because an arbitrarily oriented wave incident at this angle is reflected to become a horizontally polarized wave $[R_v(\theta) = 0]$. As the incidence angle varies from 0 to θ_0^*, the absolute value of $R_v(\theta)$ decreases from the value determined according to (23) to zero; at $\theta_0^* < \theta_0 < \pi/2$, the absolute value of $R_v(\theta)$ increases from 0 to 1. The phase $R_v(\theta)$, equal to zero at $0 < \theta_0 < \theta_0^*$, changes suddenly at $\theta_0 = \theta_0^*$ and becomes equal to π at $\theta_0^* < \theta_0 < \pi/2$.

If reflection occurs at an interface of media that satisfy the conditions $V_2/V_1 > 1$ and $\theta_0 < \theta_2$, then, for $\sin\theta_0 > V_1/V_2$, $\cos\theta_2 = \sqrt{1 - (V_2/V_1)^2 \sin^2\theta_0}$ is an imaginary value. This case corresponds to complete internal reflection from the interface of two viscous media. The angle θ_0 corresponding to the condition

$$\sin\theta_0 = V_1/V_2 \tag{26}$$

is called the angle of complete internal reflection. In this case, $\sin\theta_2 = 1$ and the refracted wave propagates parallel to the interface. Let us determine the structure of the refracted wave for incidence angles larger or equal to the limiting angle (26). In this case,

$$\cos\theta_2 = \pm i\sqrt{(V_2/V_1)^2 \sin^2\theta_0 - 1} \tag{27}$$

and, according to (15), the refracted wave has the form

$$I_T = I_2 \exp\Big(-i(\omega t - k_1 \sin\theta_0 x) - |z| k_2 \sqrt{(V_2/V_1)^2 \sin^2\theta_0 - 1}\Big).$$

This expression describes an inhomogeneous plane wave whose phase varies along the x axis and whose amplitude decreases in the z direction. Thus, in the case of complete internal reflection, an exponentially decreasing field exists in the second medium.

Figure 3 shows curves of the reflection coefficients $R_g(\theta)$ and $R_v(\theta)$ for $V_2/V_1 > 1$. According to (22) and (23), in the case of complete internal reflection, we have $|R_g| = 1$ and $|R_v| = 1$; i.e., for each components of the horizontal polarization $[R_g = |R_g| \exp(i\delta_g)]$ or vertical polarization $[R_v = |R_v| \exp(i\delta_v)]$, the intensity of the reflected defect wave is equal to the intensity of the incident wave. It is easy to calculate the variation of the reflected and incident wave phases δ_g and δ_v taking into account that expressions (22) and (23) subject to condition (27) are written as the ratio of two complex conjugate quantities:

$$\tan\frac{\delta_g}{2} = -\frac{\sqrt{(V_2/V_1)^2 \sin^2\theta_0 - 1}}{(S_1V_2/S_2V_1)\cos\theta_0}, \qquad \tan\frac{\delta_v}{2} = -\frac{\sqrt{(V_2/V_1)^2 \sin^2\theta_0 - 1}}{(S_2V_1/S_1V_2)\cos\theta_0}.$$

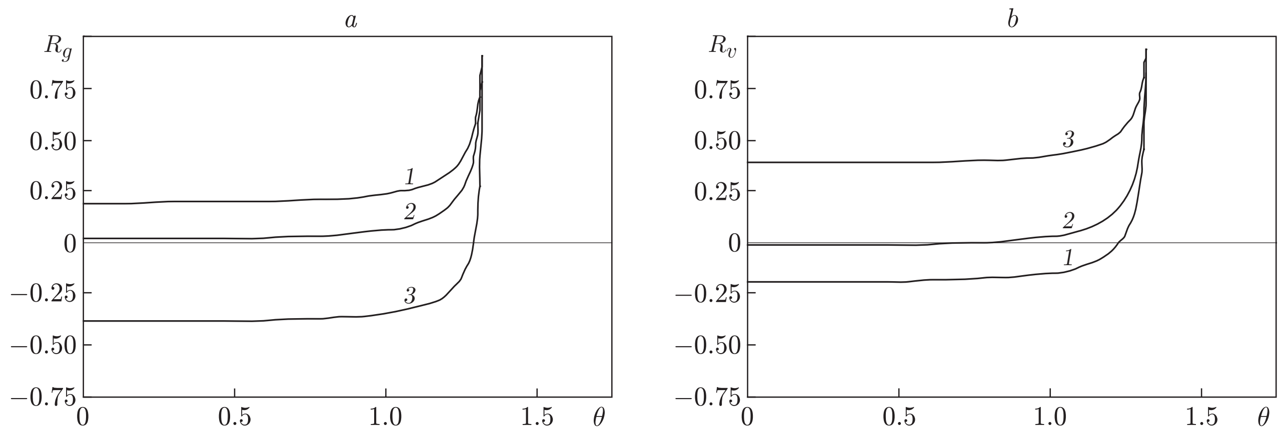

Fig. 3. Curve of $R_g(\theta)$ (a) and $R_v(\theta)$ (b) for $V_2/V_1 = 1.033$: $S_1/S_2 = 1.43$ (1), 1 (2), and 0.43 (3).

4.2. *Rapidly Decaying Waves.* As noted above, provided that $\tan\delta_1 \gg 1$ and $\tan\delta_2 \gg 1$, rapidly decaying waves propagate in the medium. The quantity

$$\frac{V_2}{V_1} = \frac{C_2}{C_1}\sqrt{\frac{1+i\tan\delta_1}{1+i\tan\delta_2}} \approx \frac{C_2}{C_1}\sqrt{\frac{\tan\delta_1}{\tan\delta_2} + \frac{i}{\tan\delta_2}\Big(\frac{\tan\delta_1}{\tan\delta_2} - 1\Big)}$$

is real only provided that one of the following conditions holds: $\tan\delta_1/\tan\delta_2 \approx 1$ or $\tan\delta_1/\tan\delta_2 \ll 1$. In these cases,

$$\frac{V_2}{V_1} \approx \frac{C_2}{C_1}\sqrt{\frac{\tan\delta_1}{\tan\delta_2}} = \sqrt{\frac{S_2\eta_1}{S_1\eta_2}}$$

all derivations of Sec. 4.1 are formally valid. However, rapidly decaying wave travel very small distances d (13) comparable with the surface layer thickness δ because $d \ll \lambda$ and $\delta \ll \lambda$; therefore, a macroscopic description is incorrect in this case.

5. General Case. For arbitrary values of $\tan\delta_1$ and $\tan\delta_2$, expression (24) is complex. In this case, the quantity θ_2 is also complex and does not have the meaning of an ordinary refraction angle (18). The coordinate-dependent part of the incident (14) and reflected (12) wave phase has the form

$$ik_1\boldsymbol{m}_0\boldsymbol{r} = i(\omega/C_1)(n_1 + i\chi_1)(x\sin\theta_0 + z\cos\theta_0). \quad (28)$$

For the refracted wave, we can similarly write

$$ik_2\boldsymbol{m}_2\boldsymbol{r} = i(\omega/C_2)(n_2 + i\chi_2)(x\sin\theta_2 + z\cos\theta_2)$$

or, using (18),

$$ik_2\boldsymbol{m}_2\boldsymbol{r} = ik_1x\sin\theta_0 + ik_2z\sqrt{1-(k_1/k_2)^2\sin^2\theta_0}. \quad (29)$$

Taking into account that

$$\cos\theta_0 = \sqrt{1-\Big(\frac{k_1}{k_2}\Big)^2\sin^2\theta_0} = \sqrt{1-\Big(\frac{C_2}{C_1}\Big)^2\Big(\frac{n_1+i\chi_1}{n_2+i\chi_2}\Big)^2\sin^2\theta_0}$$

and introducing the new representation $\cos\theta_0 = q\,\mathrm{e}^{i\gamma}$ (q and γ are real numbers), we write expression (29) in the form

$$ik_2\boldsymbol{m}_2\boldsymbol{r} = i(\omega/C_2)[(C_2/C_1)n_1x\sin\theta_0 + zq(n_2\cos\gamma - \chi_2\sin\gamma)]$$
$$- (\omega/C_2)[(C_2/C_1)\chi_1x\sin\theta_0 + zq(\chi_2\cos\gamma + n_2\sin\gamma)],$$

where

$$q^2\cos 2\gamma = 1 - (C_2/C_1)^2((n_1n_2+\chi_1\chi_2)^2 - (\chi_1n_2 - n_1\chi_2)^2)/(n_2^2+\chi_2^2)^2,$$

$$q^2 \sin 2\gamma = 2(C_2/C_1)^2((n_1 n_2 + \chi_1\chi_2)(\chi_1 n_2 - n_1\chi_2))/(n_2^2 + \chi_2^2)^2.$$

Thus, in media with arbitrary wave decay, the incident and reflected waves (28) are homogeneous because the constant amplitude surfaces and the constant phase surface defined by the equation $x \sin\theta_0 + z\cos\theta_0 = \text{const}$ coincide. The amplitude of these waves decreases by a factor of e at a distance

$$d_1 = C_1/(\chi_1\omega) = \lambda_1/(2\pi\chi_1) \tag{30}$$

in the direction of wave propagation. The refracted wave (29) is generally inhomogeneous because the constant amplitude surfaces

$$(C_2/C_1)\chi_1 x \sin\theta_0 + zq(\chi_2\cos\gamma + n_2\sin\gamma) = \text{const} \tag{31}$$

and the constant phase surfaces

$$(C_2/C_1)n_1 x \sin\theta_0 + zq(n_2\cos\gamma - \chi_2\sin\gamma) = \text{const}$$

are different. Both sets of surfaces are planes, normals to which make angles θ_a and θ_{ph} with the normal to the boundary $\boldsymbol{z}_0$, and

$$\cos\theta_a = q(\chi_2\cos\gamma + n_2\sin\gamma)/R_a, \qquad \sin\theta_a = (C_2/C_1)\chi_1\sin\theta_0/R_a,$$

$$\cos\theta_{ph} = q(n_2\cos\gamma - \chi_2\sin\gamma)/R_{ph}, \qquad \sin\theta_{ph} = (C_2/C_1)n_1\sin\theta_0/R_{ph},$$

where

$$R_a = \sqrt{((C_2/C_1)\chi_1\sin\theta_0)^2 + q^2(\chi_2\cos\gamma + n_2\sin\gamma)^2},$$

$$R_{ph} = \sqrt{((C_2/C_1)n_1\sin\theta_0)^2 + q^2(n_2\cos\gamma - \chi_2\sin\gamma)^2}.$$

Conclusions. The main results of the study are as follows. The analysis performed of the structure of the defect field waves revealed the transverse nature of the defect density and defect flux waves in viscoplastic media. The transverse nature of the defect density tensor waves was established on the basis of the kinematic identities of an elastic continuum with defects and does not depend on the material relation (3), which defines the properties of the medium. As regards the defect flux tensor waves, their nature is intimately related to the properties of the medium and the above conclusion is valid only for viscoplastic media.

In addition, the analysis of the wave structure revealed the relationship between the defect density tensor and the defect flux tensor in a plane harmonic wave. The special cases of viscoplastic media with rapid and slow wave decay were considered. The obtained conditions of rapid $[\eta/(B\omega) \gg 1]$ or slow $[\eta/(B\omega) \ll 1]$ wave decay allow one to choose conditions of surface or volumetric dynamic loading of a sample by varying the loading frequency for specified material constants.

In media with rapid wave decay, the thickness of the plastically deformed surface layer is determined by the penetration depth of defect field waves. An analytical expression is obtained that links the penetration depth of a defect field to the material characteristics and external loading parameters (13).

The reflection and refraction laws and the reflection and refraction coefficients linking the amplitudes of reflected and refracted waves to the incident wave amplitude were obtained in studying the propagation of plane defect field waves through an interface of two viscoplastic media.

Complete internal reflection and a phenomenon similar to the effect of electromagnetic wave incidence at the Brewster angle were found in analyzing the propagation of slowly decaying waves. In the case of complete internal reflection, the defect field in the refracted wave propagating along the interface of two media decays exponentially.

In media with arbitrary wave decay, the incident and reflected wave are homogeneous. The amplitudes of these waves decrease exponentially at a distance d_1 (30) in the direction of wave propagation. Probably, by varying loading conditions and the properties and geometry of the first medium, it is possible to produce conditions under which the incident wave decays without reaching the interface of two media. If the perturbation reaches the interface of two media, a refracted wave (29) arises, which is not homogeneous and decreases exponentially along the normal to the constant amplitude plane (31).

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